Appendix C

Units

In our units (the **Système International**) Coulomb's law reads

$$\mathbf{F} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{\mathscr{r}^2}\hat{\boldsymbol{\mathscr{r}}} \quad \text{(SI)}. \tag{C.1}$$

Mechanical quantities are measured in meters, kilograms, seconds, and charge is in **coulombs** (Table C.1). In the **Gaussian system**, the constant in front is, in effect, absorbed into the unit of charge, so that

$$\mathbf{F} = \frac{q_1 q_2}{\mathscr{r}^2}\hat{\boldsymbol{\mathscr{r}}} \quad \text{(Gaussian)}. \tag{C.2}$$

Mechanical quantities are measured in centimeters, grams, seconds, and charge is in **electrostatic units** (or **esu**). For what it's worth, an esu is evidently a $(\text{dyne})^{1/2}$-centimeter. Converting electrostatic equations from SI to Gaussian units is not difficult: just set

$$\epsilon_0 \rightarrow \frac{1}{4\pi}.$$

For example, the energy stored in an electric field (Eq. 2.45),

$$U = \frac{\epsilon_0}{2}\int E^2\,d\tau \quad \text{(SI)},$$

becomes

$$U = \frac{1}{8\pi}\int E^2\,d\tau \quad \text{(Gaussian)}.$$

(Formulas pertaining to fields inside dielectrics are not so easy to translate, because of differing definitions of displacement, susceptibility, and so on; see Table C.2.)

Quantity	SI	Factor	Gaussian
Length	meter (m)	10^2	centimeter
Mass	kilogram (kg)	10^3	gram
Time	second (s)	1	second
Force	newton (N)	10^5	dyne
Energy	joule (J)	10^7	erg
Power	watt (W)	10^7	erg/second
Charge	coulomb (C)	3×10^9	esu (statcoulomb)
Current	ampere (A)	3×10^9	esu/second (statampere)
Electric field	volt/meter	$(1/3) \times 10^{-4}$	statvolt/centimeter
Potential	volt (V)	$1/300$	statvolt
Displacement	coulomb/meter^2	$12\pi \times 10^5$	statcoulomb/centimeter^2
Resistance	ohm (Ω)	$(1/9) \times 10^{-11}$	second/centimeter
Capacitance	farad (F)	9×10^{11}	centimeter
Magnetic field	tesla (T)	10^4	gauss
Magnetic flux	weber (Wb)	10^8	maxwell
H	ampere/meter	$4\pi \times 10^{-3}$	oersted
Inductance	henry (H)	$(1/9) \times 10^{-11}$	second^2/centimeter

Table C.1 **Conversion Factors.** [*Note:* Except in exponents, every "3" is short for $\alpha \equiv 2.99792458$ (the numerical value of the speed of light), "9" means α^2, and "12" is 4α.]

The Biot-Savart law, which for us reads

$$\mathbf{B} = \frac{\mu_0}{4\pi} I \int \frac{d\mathbf{l} \times \hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2} \quad \text{(SI)}, \tag{C.3}$$

becomes, in the Gaussian system,

$$\mathbf{B} = \frac{I}{c} \int \frac{d\mathbf{l} \times \hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2} \quad \text{(Gaussian)}, \tag{C.4}$$

where c is the speed of light, and current is measured in esu/s. The Gaussian unit of magnetic field (the **gauss**) is the one quantity from this system in everyday use: people speak of volts, amperes, henries, and so on (all SI units), but for some reason they tend to measure magnetic fields in gauss (the Gaussian unit); the correct SI unit is the **telsa** (10^4 gauss).

One major virtue of the Gaussian system is that electric and magnetic fields have the same dimensions (in principle, one could measure the electric fields in gauss too, though no one uses the term in this context). Thus the Lorentz force law, which we have written

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \quad \text{(SI)}, \tag{C.5}$$

(indicating that E/B has the dimensions of *velocity*), takes the form

$$\mathbf{F} = q\left(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B}\right) \quad \text{(Gaussian)}. \tag{C.6}$$

In effect, the magnetic field is "scaled up" by a factor of c. This reveals more starkly the parallel structure of electricity and magnetism. For instance, the total energy stored in electromagnetic fields is

$$U = \frac{1}{8\pi}\int (E^2 + B^2)\, d\tau \quad \text{(Gaussian)}, \tag{C.7}$$

eliminating the ϵ_0 and μ_0 that spoil the symmetry in the SI formula,

$$U = \frac{1}{2}\int \left(\epsilon_0 E^2 + \frac{1}{\mu_0} B^2\right) d\tau \quad \text{(SI)}. \tag{C.8}$$

Table C.2 lists some of the basic formulas of electrodynamics in both systems. For equations not found here, and for Heaviside-Lorentz units, I refer you to the appendix of J. D. Jackson, *Classical Electrodynamics*, 3rd ed. (New York: John Wiley, 1999), where a more complete listing is to be found.[1]

[1] For an interesting "primer" on electrical SI units see N. M. Zimmerman, *Am. J. Phys.* **66**, 324 (1998).

	SI	Gaussian
Maxwell's equations		
In general:	$\left\{\begin{array}{l} \nabla\cdot\mathbf{E}=\frac{1}{\epsilon_0}\rho \\ \nabla\times\mathbf{E}=-\partial\mathbf{B}/\partial t \\ \nabla\cdot\mathbf{B}=0 \\ \nabla\times\mathbf{B}=\mu_0\mathbf{J}+\mu_0\epsilon_0\partial\mathbf{E}/\partial t \end{array}\right.$	$\begin{array}{l} \nabla\cdot\mathbf{E}=4\pi\rho \\ \nabla\times\mathbf{E}=-\frac{1}{c}\partial\mathbf{B}/\partial t \\ \nabla\cdot\mathbf{B}=0 \\ \nabla\times\mathbf{B}=\frac{4\pi}{c}\mathbf{J}+\frac{1}{c}\partial\mathbf{E}/\partial t \end{array}$
In matter:	$\left\{\begin{array}{l} \nabla\cdot\mathbf{D}=\rho_f \\ \nabla\times\mathbf{E}=-\partial\mathbf{B}/\partial t \\ \nabla\cdot\mathbf{B}=0 \\ \nabla\times\mathbf{H}=\mathbf{J}_f+\partial\mathbf{D}/\partial t \end{array}\right.$	$\begin{array}{l} \nabla\cdot\mathbf{D}=4\pi\rho_f \\ \nabla\times\mathbf{E}=-\frac{1}{c}\partial\mathbf{B}/\partial t \\ \nabla\cdot\mathbf{B}=0 \\ \nabla\times\mathbf{H}=\frac{4\pi}{c}\mathbf{J}_f+\frac{1}{c}\partial\mathbf{D}/\partial t \end{array}$
D and H		
Definitions:	$\left\{\begin{array}{l} \mathbf{D}=\epsilon_0\mathbf{E}+\mathbf{P} \\ \mathbf{H}=\frac{1}{\mu_0}\mathbf{B}-\mathbf{M} \end{array}\right.$	$\begin{array}{l} \mathbf{D}=\mathbf{E}+4\pi\mathbf{P} \\ \mathbf{H}=\mathbf{B}-4\pi\mathbf{M} \end{array}$
Linear media:	$\left\{\begin{array}{ll} \mathbf{P}=\epsilon_0\chi_e\mathbf{E}, & \mathbf{D}=\epsilon\mathbf{E} \\ \mathbf{M}=\chi_m\mathbf{H}, & \mathbf{H}=\frac{1}{\mu}\mathbf{B} \end{array}\right.$	$\begin{array}{ll} \mathbf{P}=\chi_e\mathbf{E}, & \mathbf{D}=\epsilon\mathbf{E} \\ \mathbf{M}=\chi_m\mathbf{H}, & \mathbf{H}=\frac{1}{\mu}\mathbf{B} \end{array}$
Lorentz force law	$\mathbf{F}=q(\mathbf{E}+\mathbf{v}\times\mathbf{B})$	$\mathbf{F}=q\left(\mathbf{E}+\frac{\mathbf{v}}{c}\times\mathbf{B}\right)$
Energy and power		
Energy:	$U=\frac{1}{2}\int(\epsilon_0E^2+\frac{1}{\mu_0}B^2)\,d\tau$	$U=\frac{1}{8\pi}\int\left(E^2+B^2\right)\,d\tau$
Poynting vector:	$\mathbf{S}=\frac{1}{\mu_0}(\mathbf{E}\times\mathbf{B})$	$\mathbf{S}=\frac{c}{4\pi}(\mathbf{E}\times\mathbf{B})$
Larmor formula:	$P=\frac{1}{4\pi\epsilon_0}\frac{2}{3}\frac{q^2a^2}{c^3}$	$P=\frac{2}{3}\frac{q^2a^2}{c^3}$

Table C.2 **Fundamental Equations in SI and Gaussian Units.**